3G rule for attending in person lectures at KIT:

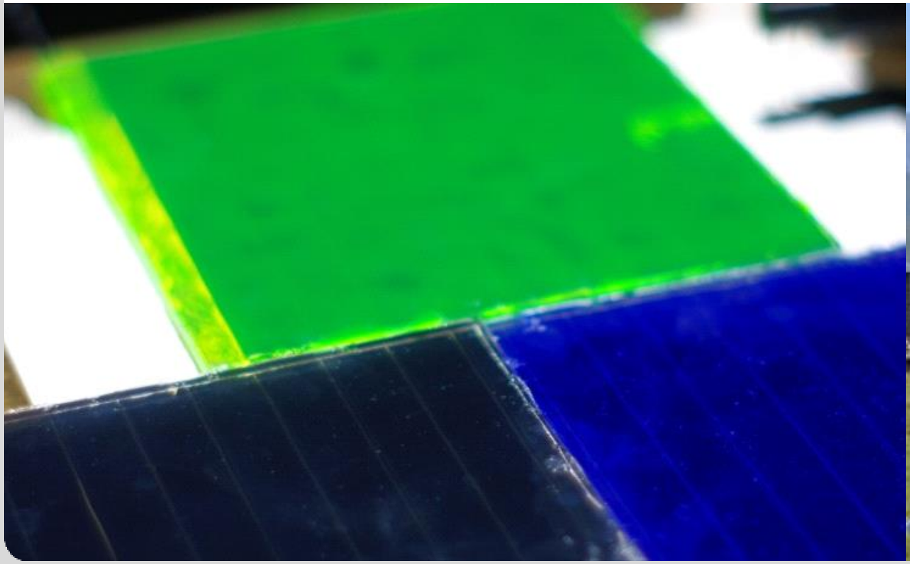
- ***geimpft* – vaccinated**
- ***genesen* – recovered**
- ***getestet* – tested**

“Solar Energy” WS 2021/2022

Lecture 3: Basics of Electrodynamics & Semiconductor Physics

Tenure-Track-Prof. Dr. Ulrich W. Paetzold

Institute of Microstructure Technology (IMT), Hermann-von-Helmholtz-Platz 1, 76344 Eggenstein-Leopoldshafen
Light Technology Institute (LTI), Engesserstrasse 13, Building 30.34, 76131 Karlsruhe



Information

Karlsruher Institut für Technologie

“Solar Energy“ lecture (23745) and tutorials (23750)

Lecturers: **Prof. Dr. Bryce Richards**

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URL: http://www.imt.kit.edu/richards.php

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URL: http://www.imt.kit.edu/richards.php

Information

- Tutors: Benni Hacene (benjamin.hacene@kit.edu)
  Emmanuel Ogunniyi (emmanuel.ogunniyi@kit.edu)

- Slides and further information will be available as PDF files for download at IIAS https://ilias.studium.kit.edu
- Six tutorial sessions throughout the course
- One lab demonstration
- One excursion – to the KIT Solar Park (1MW PV installation)
- Exam will be a 2 hour written exam, **Wed 08th March 2022**

“Solar Energy” lecture plan – WS 2020-2021 – Prof. Bryce Richards & T.T.Prof. Uli Paetzold

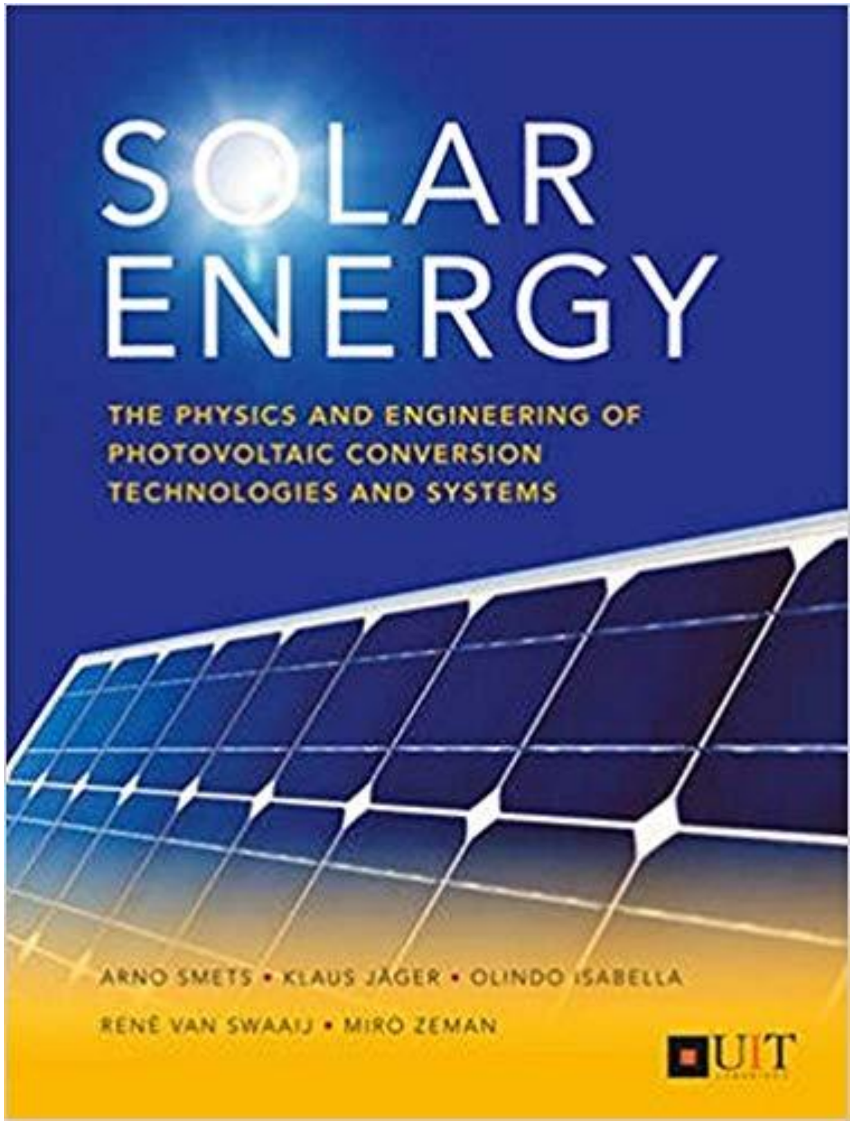
LTI Hörsaal, Geb. 30.34, Monday 10:00 – 11:30	LTI Hörsaal, Geb. 30.34, Thursday 12:00 – 13:30
Mon 18 Oct 2021	Thu 21 Oct 2021
Lecture 1: Introduction (Chpt 1 – 3) (Prof. Richards)	Lecture 2: Solar Radiation (Chpt 5) (Prof. Richards)
Mon 25 Oct 2021	Thu 28 Oct 2021
Lecture 3: Basics of Electrodynamics & Semiconductor Physics (Chpt. 4 & 6) (Prof. Paetzold)	Lecture 4: Generation & Recombination (Chpt 7) (Prof. Paetzold)
Mon 1 Nov 2021	Thu 4 Nov 2021
NO LECTURE or TUTORIAL	Tutorial 1: Chpts 1 – 6
Mon 8 Nov 2021	Thu 11 Nov 2021
Lecture 5: Semiconductor Junctions (Chpt 8) (Prof. Paetzold)	Lecture 6: Solar Cell Parameters (Chpt 9) (Prof. Paetzold)
Mon 15 Nov 2021	Thu 18 Nov 2021
Lecture 7: Crystalline Silicon Solar Cells (Chpt 11 & 12) (Prof. Richards)	Tutorial 2: Chpts 7 – 9
Mon 22 Nov 2021	Thu 25 Nov 2021
Lecture 8: Thin-Film Solar Cells: inorganic (Chpt 13) (Prof. Richards)	Lecture 9: Thin-Film Solar Cells: OPV/perovskite (Chpt 13) (Prof. Paetzold)
Mon 29 Nov 2021	Thu 2 Dec 2021
Lecture 10: PV Modules (Chpt 15) (Prof. Richards)	Tutorial 3: Chpts 11 – 13
Mon 6 Dec 2021	Thu 9 Dec 2020
Lecture 11: PV Systems & Components (Chpt 17 & 19) (Prof. Richards)	Lecture 12: Location Issues + PV System Design + PV in developing countries (Chpt 18 & 20) (Prof. Richards)

Mon 13 Dec 2021	Thu 16 Dec 2020
Lecture 13: PV System Economics & Environmental Aspects (Chpt 21) (Prof. Richards)	Tutorial 4: Chpts 15, 17, 19
Mon 20 Dec 2021	Thu 23 Dec 2021
NO LECTURE or TUTORIAL	*NO LECTURE or TUTORIAL*
Mon 27 Dec 2021	Thu 30 Dec 2021
NO LECTURE or TUTORIAL	*NO LECTURE or TUTORIAL*
Mon 3 Jan 2022	Thu 6 Jan 2022
NO LECTURE or TUTORIAL	*NO LECTURE or TUTORIAL*
Mon 10 Jan 2022	Thu 13 Jan 2022
Demo: Getting hands-on with solar cells (Bahram Abdollahi)	Lecture 14: Spectral conversion for PV (extra) (Prof. Richards)
Mon 17 Jan 2022	Thu 20 Jan 2022
Tutorial 5: Chpts 18 & 20-21	Lecture 15: Light management for PV (Chpt 10 + extra) (Prof. Paetzold)
Mon 24 Jan 2022	Thu 27 Jan 2022
Lecture 16: Losses, Efficiency Limits and Third Gen. Concepts (Chpts 10 & 16) (Prof. Paetzold)	Lecture 17: Tandem Solar Cells (extra) (Prof. Paetzold)
Mon 31 Jan 2022	Thu 3 Feb 2022
Tutorial 6: Chpt 10 + 16 + extra	EXCURSION Visit to KIT Solar Park (Prof. Richards)
Mon 7 Feb 2022	Thu 10 Feb 2022
Mock exam	Review and Q&A session (Prof. Paetzold & Prof. Richards)

Textbook

- “*Solar Energy: The physics and engineering of photovoltaic conversion, technologies and systems*” (2016)

  by Smets, Jäger, Isabella, van Swaaij (TU Delft)
- Online: €30.50 (paperback)
  €0.00 (Kindle)
- Technological focus
- Lectures largely follow chapters in textbook:
  - Some chapters skipped
  - Some material supplemented
  - Data updated



SOLAR ENERGY

THE PHYSICS AND ENGINEERING OF PHOTOVOLTAIC CONVERSION TECHNOLOGIES AND SYSTEMS

ARNO SMETS • KLAUS JÄGER • OLINDO ISABELLA
RENÉ VAN SWAAIJ • MIRO ZEMAN

UIT

Source: textbook

Electrodynamics (Optics) Basics

Karlsruher Institut für Technologie

The electromagnetic theory: Electricity and magnetism have been known since ancient times, but only in the 19th these phenomena were linked.

- in 1861/1862, the Scottish physicist James C. Maxwell published the electromagnetic theory by a set of equations, the Maxwell equations:

(M1) $\vec{\nabla} \cdot \vec{D} = \rho$

(M2) $\vec{\nabla} \cdot \vec{B} = 0$

(M3) $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(M4) $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{j}$

Maxwell

where **r** and *t* denote location and time, respectively. **D** is the *electric displacement*, **E** the *electric field*, **B** the *magnetic induction* and **H** the *magnetic field*. *ρ* is the *free charge density* and **J** the *free current density*.

- Electric displacement and field as well as magnetic field and induction are linked (∈ = relative permittivity *μ* = relative permeability of the medium)

$$\vec{D} = \epsilon_0 \epsilon(\mathbf{r})\, \vec{E} \qquad \vec{B} = \mu_0 \mu(\mathbf{r})\, \vec{H}$$

Constants defined in vacuum: $\epsilon_0 = 8.85\times10^{-12}$ As/(Vm), $\mu_0 = 4\pi\times10^{-7}$ Vs/(Am).

Source: textbook

Electrodynamics (Optics) Basics

One of the most important predictions of the *Maxwell equations* is the presence of electromagnetic waves!

- Consider an isotropic, source-free ($\rho = 0$, **j** = 0), non-magnetic ($\mu = 1$) space.
- Applying the rotation operator ∇ × to the 2nd Maxwell equation (M2):

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}\vec{\nabla} \times B \xrightarrow{\text{(M4,*, M3)}} \Delta\vec{E} = \epsilon_0 \epsilon\, \mu_0 \mu \frac{\partial^2}{\partial t^2}\vec{E}$$

*using: $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = \vec{\nabla}\left(\vec{\nabla} \times \vec{E}\right) - \Delta\vec{E}$ with $\vec{\nabla}\left(\vec{\nabla} \times \vec{E}\right) = 0$;

- Using further c= 1/$\epsilon_0\mu_0$ and n² = ϵ leads to the Helmholtz Wave Equations:

$$\Delta\vec{E} - \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} \qquad\qquad \Delta\vec{H} - \frac{n^2}{c^2}\frac{\partial^2\vec{H}}{\partial t^2} \quad **$$

** In a similar manner we can derive the wave equation for $\vec{H}$

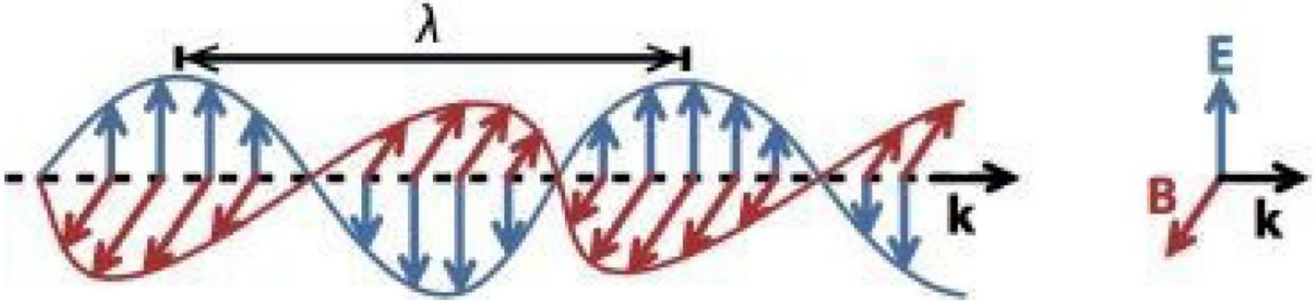
Source: textbook

Electrodynamics (Optics) Basics

The simplest solution to *Helmholtz Wave Equations* is the plane harmonic wave, where light of constant wavelength *λ* propagates in one direction:

$$\vec{E}(\vec{r},t) = \vec{E}_0 \times e^{ik_z z - i\omega t} \qquad \vec{H}(\vec{r},t) = \vec{H}_0 \times e^{ik_z z - i\omega t}$$

where $\vec{E}_0$ and $\vec{H}_0$ are constant vectors (the amplitudes), k_z is the wave number and ω is the angular frequency.



Note, k_z and ω are connected, which implies several useful interrelations:

$$k_z^2 - \frac{n^2}{c^2}\omega^2 = 0 \Rightarrow \lambda = \frac{2\pi}{k_z} = \frac{2\pi c}{n\omega}$$

Source: textbook

Electrodynamics (Optics) Basics

Light is also a particle, not only a wave!

- It was Einstein, who understood in 1905 that the energy transfer between a photon and another particle (i.e. with the photoelectric effect to an electron) is not determined by amplitudes of the EM wave ($\vec{E}_0$ and $\vec{H}_0$), but the quantized energy of the photon.

$$E_{ph} = hv$$

- Photons carry a quantized amount of energy as well as momentum.
- Interestingly the famous Planck constant *h* appeared first earlier in Planck’s law that describes the spectral density of electromagnetic radiation emitted by a black body in thermal equilibrium at a given temperature T, when there is no net flow of matter or energy between the body and its environment (see last lecture).

$$M_\lambda^0(\lambda,T) = \frac{2\pi hc^2}{\lambda^5} \cdot \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1}$$

Source: textbook

A humoristic perspective... *What is light?*

“In the beginning God created the heaven and the earth.
And the earth was without form, and void;
and darkness was upon the face of the deep.
And the Spirit of God moved upon the face of the waters.
And God said, Let there be light: and there was light.” Genesis, Chapter 1

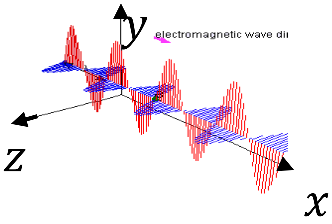
But what God really said was much more complex ☺:

Light is a wave ...

Maxwell

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{D}=\rho \qquad \vec{\nabla}\times\vec{H}=\frac{\partial\vec{D}}{\partial t}+\vec{j}$$



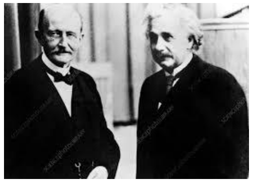
Wave properties:
EM radiation has a wave like behavior

... ‚but light is also a particle

$$E_{ph}=h\nu$$

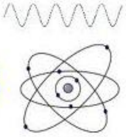
$$p_{ph}=\frac{h}{2\pi}k$$

$$E_{ph}=p_{ph}c$$



Plank, Einstein

Photons
Electrons



Particle properties:
Light is emitted and absorbed as discrete packets of energy,
i.e. quanta, called photons

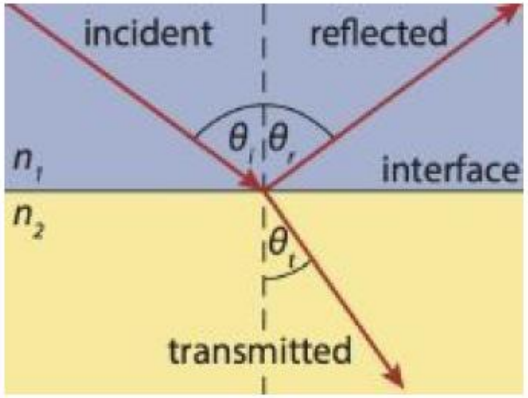
Optics of flat interfaces

Karlsruher Institut für Technologie

Electrodynamics explain optics of flat interfaces

- Snell‘s law of refraction (assuming no absorption):

$$n_1 \sin\theta_\mathrm{i} = n_2 \sin\theta_\mathrm{t}\,.$$



- ..can be derived in many ways, e.g. by applying the wave solution of Helmholtz Wave Equation for the reflected, transmitted and incident waves to the fundamental boundary conditions at the interface between two materials (assuming Re(n) = n).

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0 \qquad \vec{n} \times (\vec{H}_2 - \vec{H}_1) = 0$$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = 0 \qquad \vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

Source: textbook

Optics of flat interfaces

- Relations between the magnitudes of the incident, reflected and refracted fields are described by Fresnel equations that are derived using fundamental boundary conditions at the interface between two materials.

- For *perpendicular (s = senkrecht)* and parallel *(p=parallel)* polarized light the Fresnel equations that describe the amplitudes are given by

$$t_s = \left(\frac{\xi_{0t}}{\xi_{0i}}\right)_s = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}, \qquad t_p = \left(\frac{\xi_{0t}}{\xi_{0i}}\right)_p = \frac{2n_1\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i},$$

$$r_s = \left(\frac{\xi_{0r}}{\xi_{0i}}\right)_s = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}, \qquad r_p = \left(\frac{\xi_{0r}}{\xi_{0i}}\right)_p = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i}.$$

- The reflectivity is given by considering that intensities are proportional to the square of the electric field:

$R = \frac{1}{2}\left(r_s^2 + r_p^2\right).$ and for normal incidence $R(\theta_i = 0) = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$

- Considering further conservation of energy: $T(\theta_i = 0) = \frac{4n_1 n_2}{(n_1 + n_2)^2}$

Source: textbook

Optics in absorptive media

- In general, the optical properties of an absorbing medium are described by a *complex electric permittivity* $\varepsilon = \varepsilon' + \mathrm{i}\,\varepsilon''$.
- Since the refractive index is given as the square root of $\mathrm{n} = \sqrt{\varepsilon}$, it is complex too, $\mathrm{n} = \mathrm{n} + \mathrm{i}\,\kappa$ (*κ* denotes the imaginary part)
- As a consequence, the wave vector is complex $\mathrm{k_z} = k_z' + \mathrm{i}k_z''$ and the Electric field intensity is attenuated exponentially:

$$\vec{E}(\vec{r},t) = \vec{E}_0\, e^{ik_z z - i\omega t} = \vec{E}_0 e^{-k_z''} e^{ik_z' z - i\omega t}$$

- As a consequence, the intensity attunats exponentially as described by the Lambert-Beer Law:

$$I(\vec{r}) = |\vec{E}(\vec{r},t)| = I_0\, e^{-\alpha z}.$$

Source: textbook

QUESTIONS ?

Semiconductors

<u>What are typical/established semiconductors materials?</u>

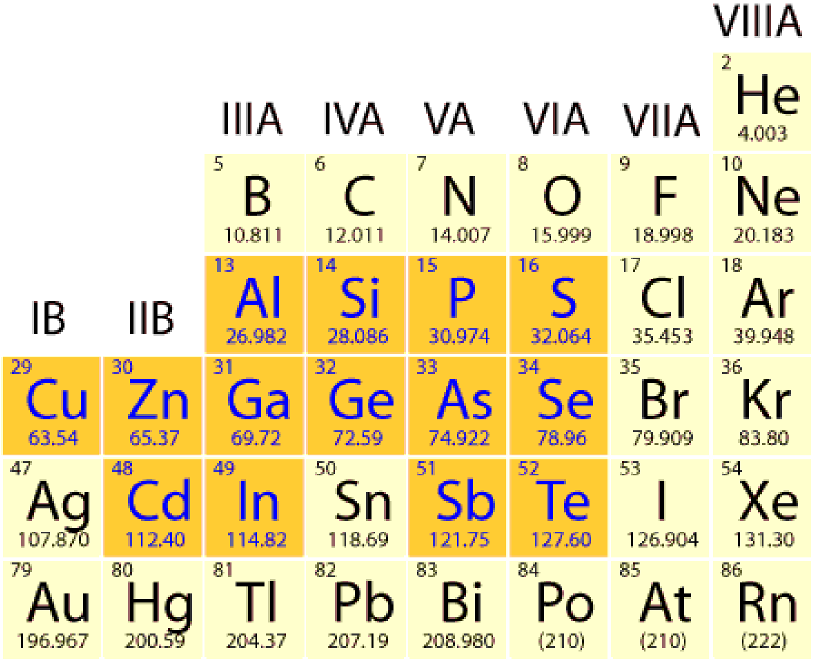
The most common semiconductors are inorganic materials typically from either group IV of the periodic table, or combination of III-V or II-VI

As semiconductors made from different elements the properties vary

Silicon (group IV)

- most commonly used semiconductor material as it forms the basis for integrated circuit (IC) industry
- most mature technology
- most solar cells are Si based

∴ Si will be used as a model material over next few lectures



IB	IIB	IIIA	IVA	VA	VIA	VIIA	VIIIA
							2 He 4.003
		5 B 10.811	6 C 12.011	7 N 14.007	8 O 15.999	9 F 18.998	10 Ne 20.183
		13 Al 26.982	14 Si 28.086	15 P 30.974	16 S 32.064	17 Cl 35.453	18 Ar 39.948
29 Cu 63.54	30 Zn 65.37	31 Ga 69.72	32 Ge 72.59	33 As 74.922	34 Se 78.96	35 Br 79.909	36 Kr 83.80
47 Ag 107.870	48 Cd 112.40	49 In 114.82	50 Sn 118.69	51 Sb 121.75	52 Te 127.60	53 I 126.904	54 Xe 131.30
79 Au 196.967	80 Hg 200.59	81 Tl 204.37	82 Pb 207.19	83 Bi 208.980	84 Po (210)	85 At (210)	86 Rn (222)

Source: *http://www.pveducation.org/pvcdrom/pn-junction/semiconductor-materials*

Silicon

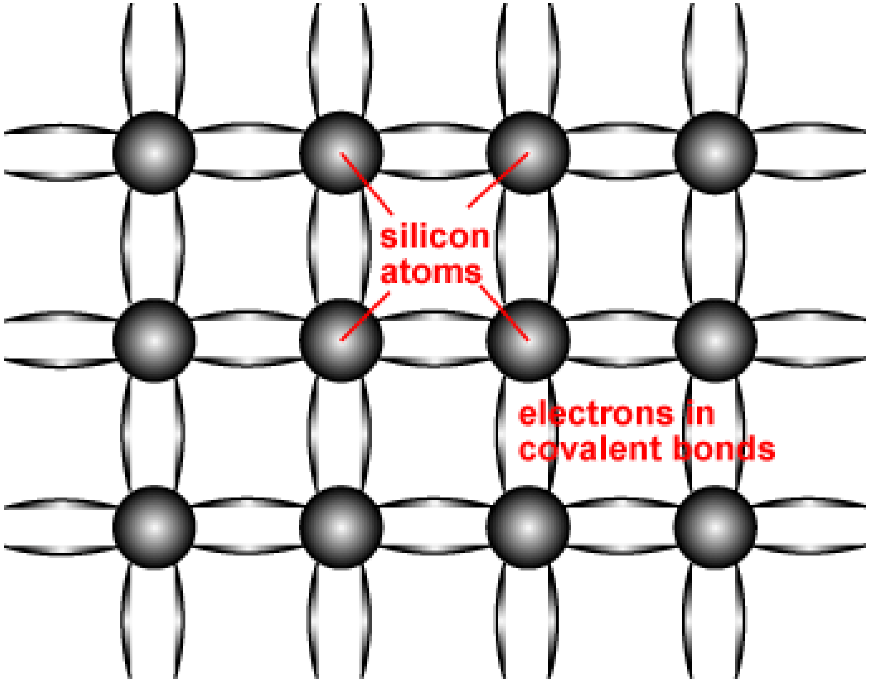
Silicon – individual atoms bonded together in a periodic structure so that each atom is surrounded by 8 electrons.

Si atom – nucleus contains a core of protons (positively charged particles) and neutrons (particles having no charge)
surrounded by electrons.

Number of electrons (e^-) = protons so as atom is overall electrically neutral

Electrons form a covalent bond – two atoms sharing a single e^- (eight overall between four covalent bonds)

Bond structure determines the material properties of a semiconductor



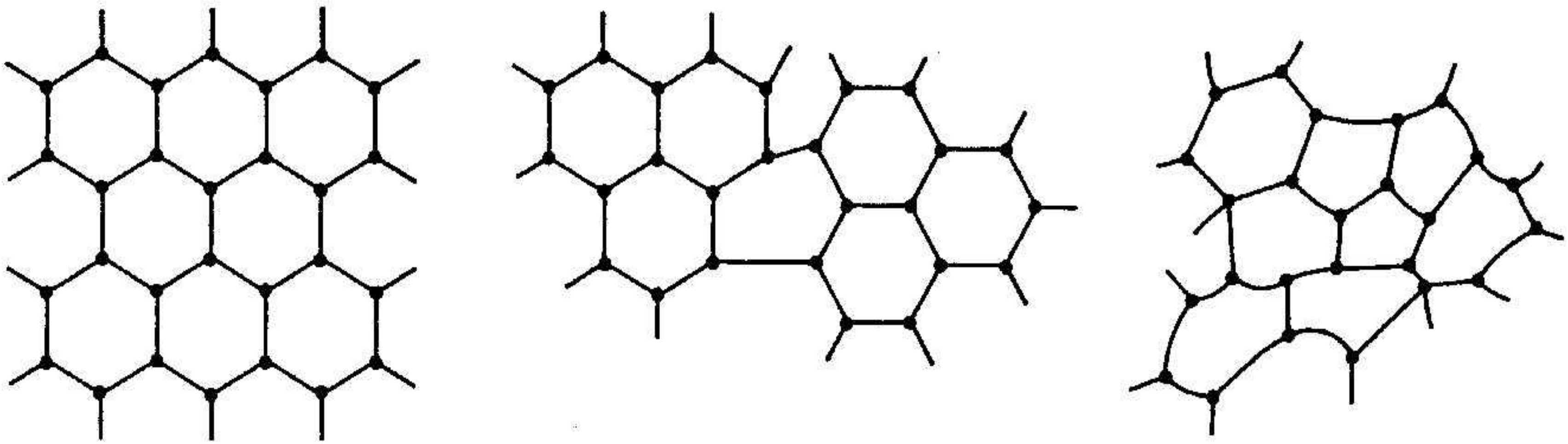
Source: http://www.pveducation.org/pvcdrom/pn-junction/semiconductor-structure

Source: textbook

Silicon

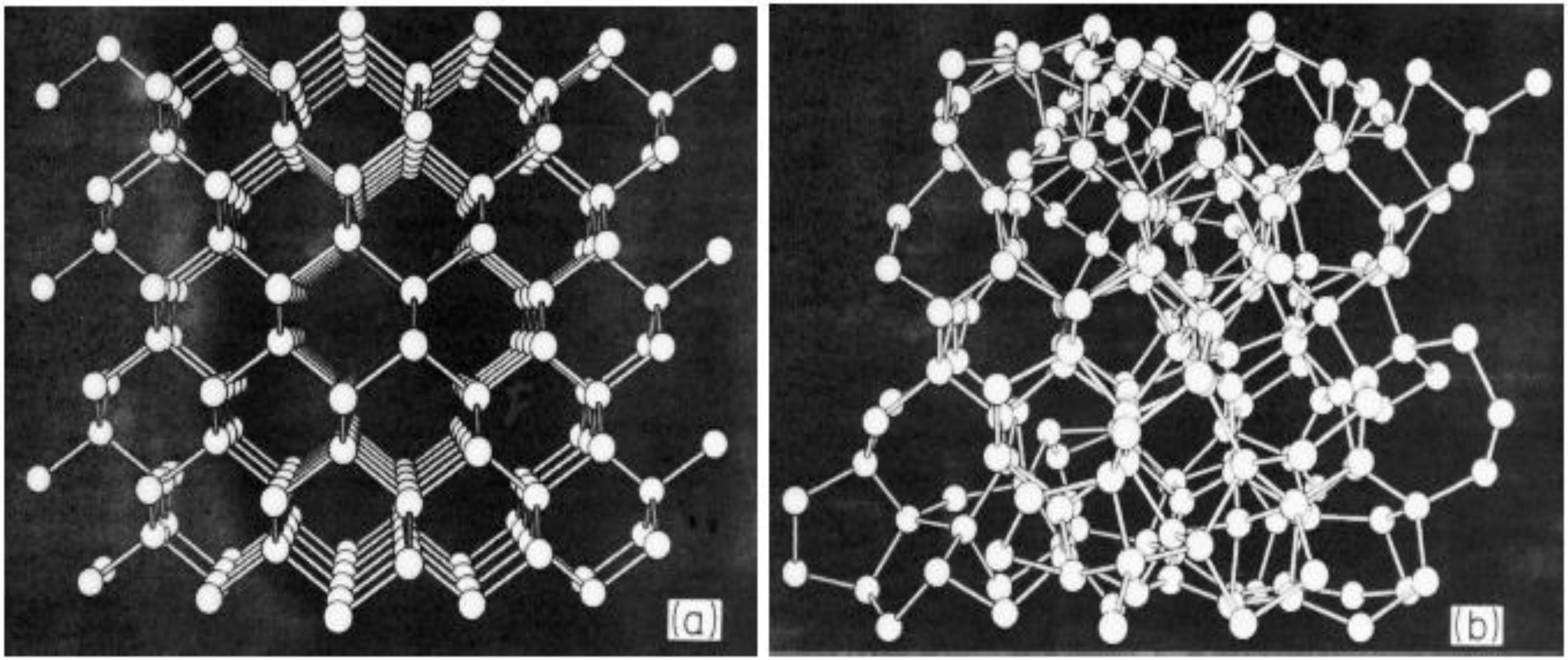
Silicon – depending on fabrication technique can get different material structure that affect properties greatly:

- Single crystal (c-Si): atoms are periodically arranged in a lattice
- Poly- or multi-crystalline (mc-Si): there are regions of crystallinity but no long range order
- Amorphous silicon (a-Si): only short range order exists and no periodicity



Source: Wolfe, Holonyak, Stillman

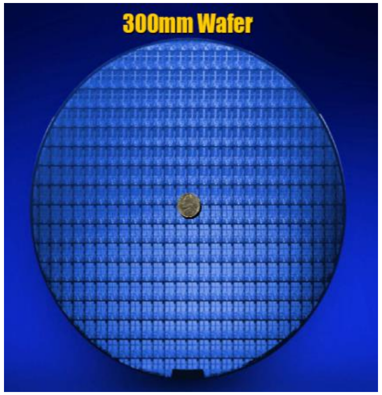
Silicon



c-Si

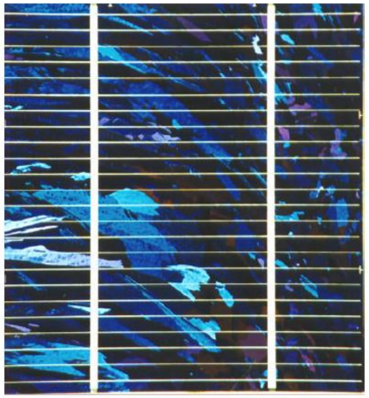
a-Si

Silicon

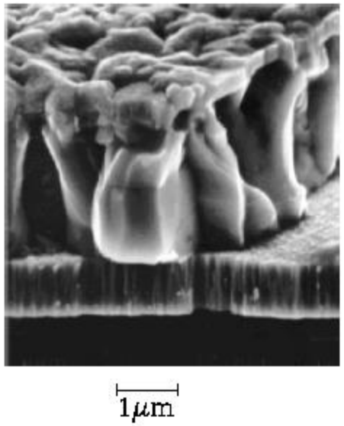


300 mm wafer
for Si microelectronics

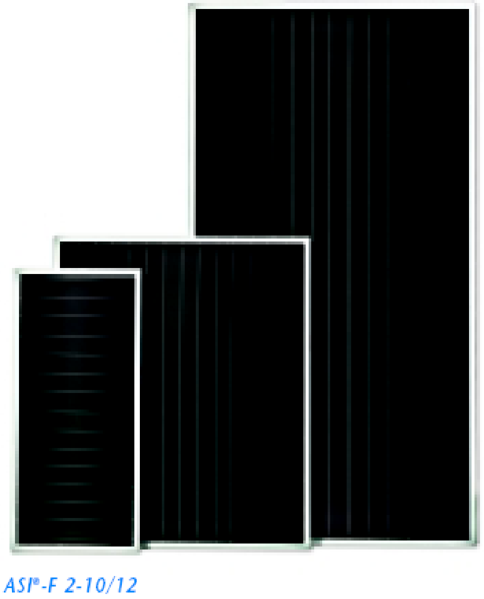
Monocrystalline
Si solar cell



Multicrystalline
Si solar cell



Electron microscope image
of thin film solar cell

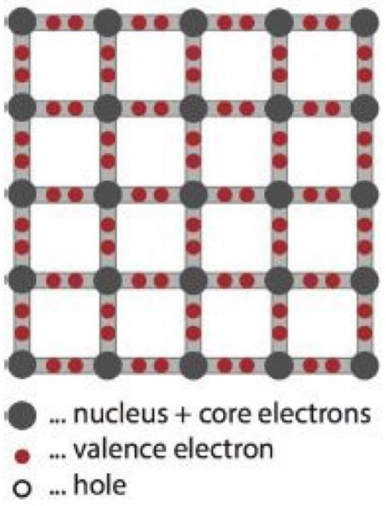


PV modules made
from thin film
amorphous
silicon

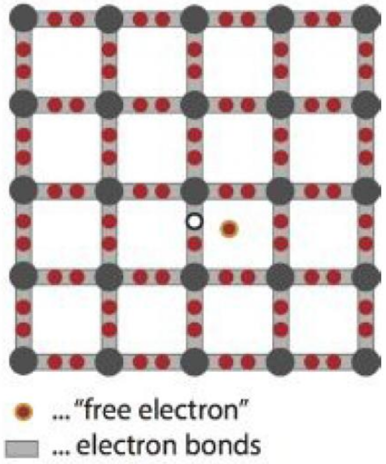
Intrinsic semiconductor

- When the concentration of impurity atoms in a semiconductor is insignificant it is called an *intrinsic semiconductor*.
- At T > 0 K the bonds start to break due to the absorption of thermal energy.
- This process results in the creation of mobile electrons and holes. In intrinsic semiconductors the concentration of electrons is equal to the concentration of holes.
- At 300 K around 1.5×10^{10} cm^{-3} broken bonds are present. This number then also gives the concentration of holes, p, and electrons. It is referred to as the intrinsic carrier concentration and is denoted as n_i.
- Note, the density of Si atoms is several orders of magnitude higher 5×10^{22} cm^{-3}.

T = 0K



T = 300K



Source: textbook

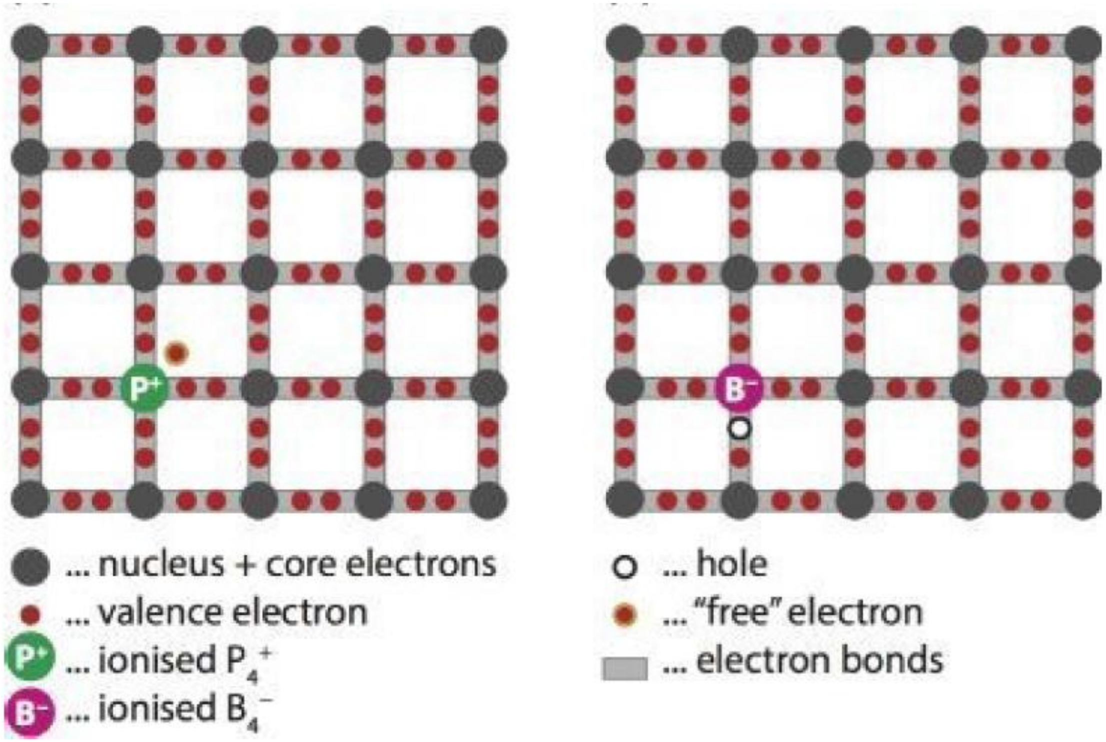
Doping

Change balance of e^- and h^+ in Si crystal lattice by doping with other atoms

- group V elements have one more valence e^- than Si
  $\Rightarrow$ produce "*n*-type" semiconductor material
- group III elements have one less valence e^- than Si
  $\Rightarrow$ produce “*p*-type" semiconductor material

In doped material, there is always more of one type of carrier than the other.

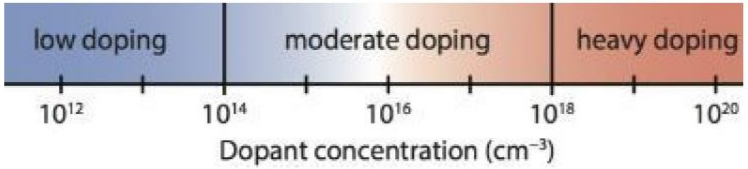
Carrier with
higher concentration
= "majority carrier"
while other
= "minority carrier"



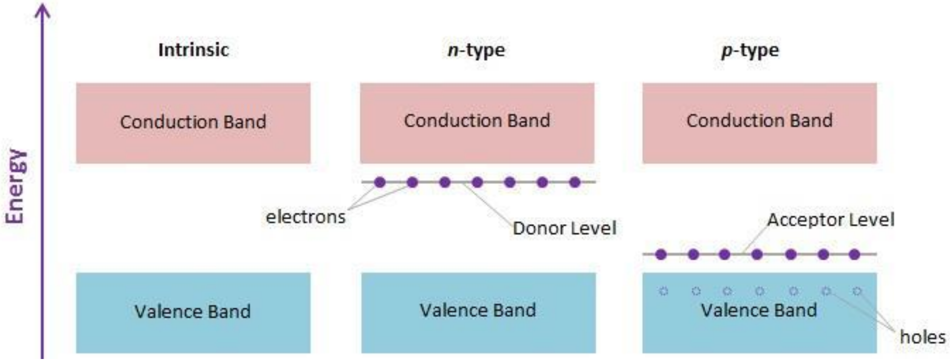
Source: textbook

Doping

- The most used elements to dope c-Si are boron (B) and phosphorus (P), with atomic numbers of 5 and 15, respectively.
- The P atom *“donates”* a free (mobile) electron into the c-Si lattice. The impurity atoms are called *donors*. N_D = concentration of donors;
- Contrary, the B atom induces a hole that can move around the lattice. The impurity atom is called acceptors. N_A = concentration of donors.
- Lightly and moderately doped semiconductors are referred to as extrinsic. A semiconductor doped to such high levels that it acts more like a conductor than a semiconductor is referred to as degenerate.



The range of doping levels used in c-Si.

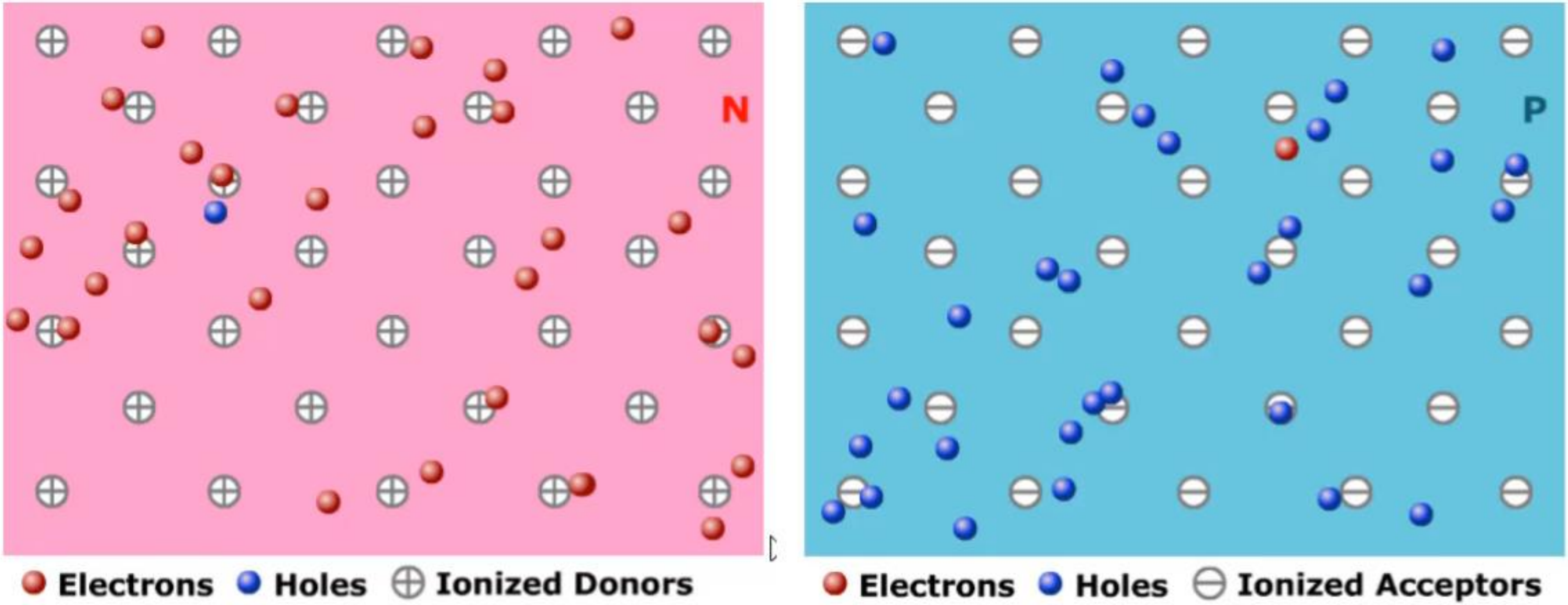


Source: textbook

Doping

n-type Si (majority carriers are **N**egatively charged electrons)
p-type Si (majority carriers are **P**ositively charged holes)

e.g. 10^{17} cm^{-3} majority carriers and 10^{6} cm^{-3} minority carriers



Source: *http://www.pveducation.org/pvcdrom/pn-junction/doping*

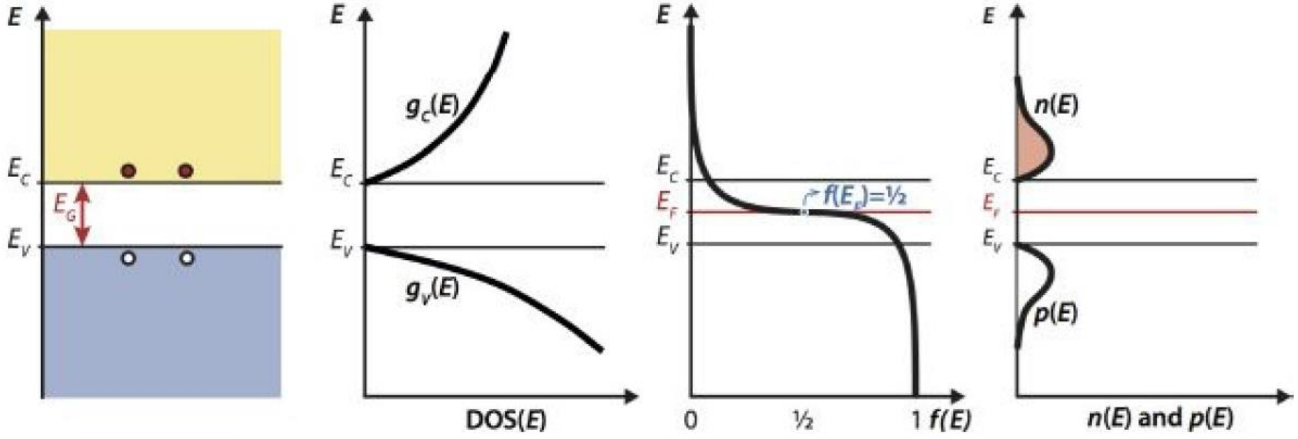
QUESTIONS ?

Carrier concentrations

Intrinsic semiconductor in thermal equilibrium:

The carrier concentrations n(E) and p(E) are a product of:

1. The *density of states* function DoS(*E*) per unit volume and energy. Given for conduction band (CB) close to *EC* and in the valence band (VB) close to *EV* by $g_c(E)$ and $g_v(E)$, respectively.
2. The occupation function is the Fermi–Dirac distribution function, *f* (*E*), which describes the ratio of states filled with an electron to total allowed states at given energy *E*.



$$n(E) = g_C(E) f(E),$$

$$p(E) = g_V(E)\,[1 - f(E)]$$

$$n = \int_{E_C}^{E_{top}} n(E)\,dE,$$

$$p = \int_{E_{bottom}}^{E_V} p(E)\,dE.$$

Source: textbook

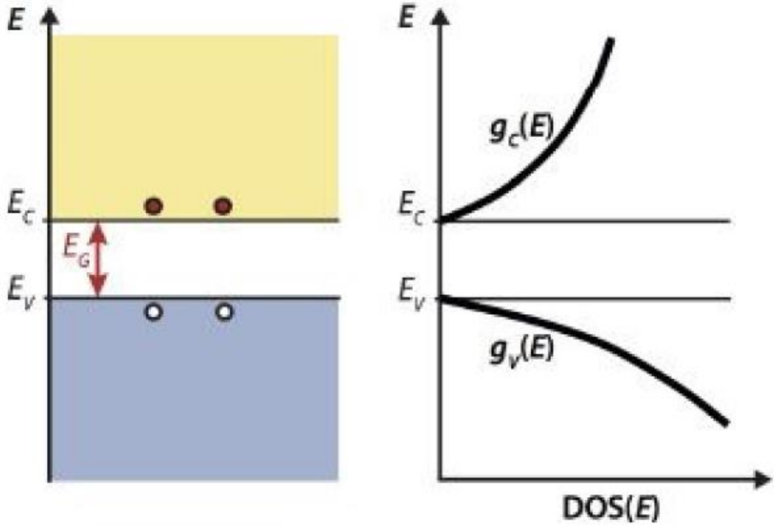
Carrier concentrations

<u>Intrinsic semiconductor in thermal equilibrium:</u>

1. The *density of states* function DoS(*E*) can be approximated for the conduction band close to E_C and in the valence band close to E_V by:

$$g_C(E) = 4\pi \left(\frac{2m_n^*}{h^2}\right)^{\frac{3}{2}} \sqrt{E - E_C},$$

$$g_V(E) = 4\pi \left(\frac{2m_p^*}{h^2}\right)^{\frac{3}{2}} \sqrt{E - E_V},$$



where m_n^* and m_p^* are the *effective masses* of electrons and holes, respectively. The effective mass takes the effect of a periodic force of the crystal lattice as well as anisotropy into account.

Source: textbook

Carrier concentrations

Intrinsic semiconductor in thermal equilibrium:

2. The Fermi–Dirac distribution function, *f* (*E*), describes the ratio of states filled with an electron to total allowed states at given energy *E*.

$$f(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_B T}\right)},$$

k_B is the Boltzmann's constant ($k_B = 1.38 \times 10^{-23}$ J/K) and E_F is the *Fermi energy/level,* which is the electrochemical potential of the electrons in a material. It represents the averaged energy of electrons in the material.

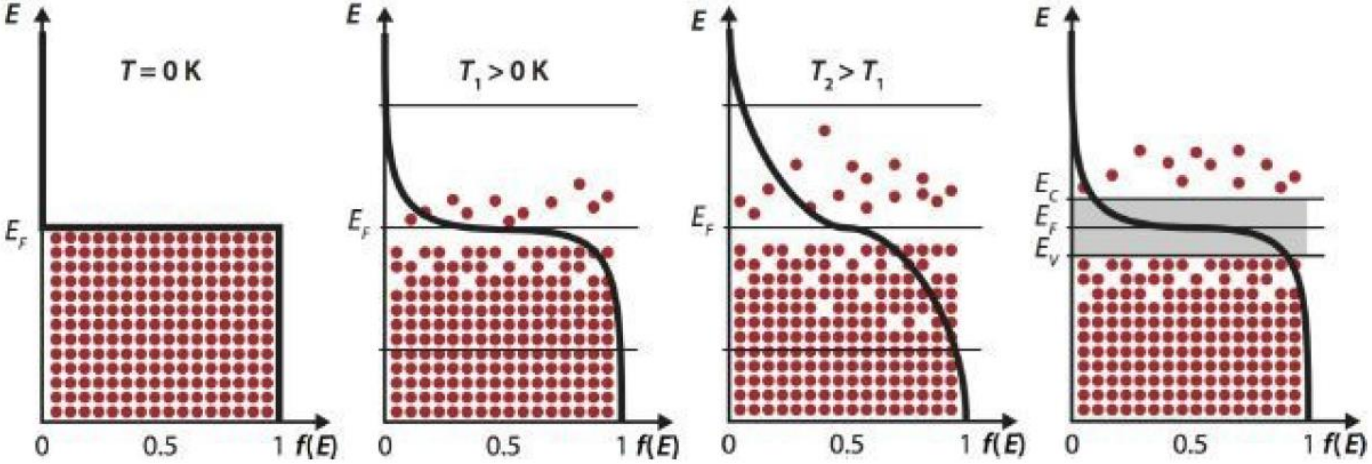


Figure 6.6: The Fermi–Dirac distribution function. (a) For $T = 0$ K, all allowed states below the Fermi level are occupied by two electrons. (b, c) At $T > 0$ K not all states below the Fermi level are occupied and there are some states above the Fermi level that are occupied. (d) In an energy gap between bands no electrons are present.

Source: textbook

Carrier concentrations

Intrinsic semiconductor in thermal equilibrium:

The total charge carrier density is then given by:

$$n = \int_{E_C}^{E_{\mathrm{top}}} n(E)\mathrm{d}E, \quad p = \int_{E_{\mathrm{bottom}}}^{E_V} p(E)\mathrm{d}E.$$

By inserting the DOS(E) and f(E), we derive:

$$n = N_C \exp\left(\frac{E_F - E_C}{k_B T}\right) \qquad \text{for} \qquad E_C - E_F \geq 3k_B T,$$

$$p = N_V \exp\left(\frac{E_V - E_F}{k_B T}\right) \qquad \text{for} \qquad E_F - E_V \geq 3k_B T,$$

where N_C and N_V are the effective densities of the conduction band states and the valence band states, respectively. They are defined as

$$N_C = 2\left(\frac{2\pi m_n^* k_B T}{h^2}\right)^{\frac{3}{2}} \qquad \text{and} \qquad N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{\frac{3}{2}}$$

For crystalline silicon at 300 K: $N_C = 3.2\ 10^{19}$ cm^{-3} and $N_V = 1.8\ 10^{19}$ cm^{-3}

Source: textbook

Carrier concentrations

Intrinsic semiconductor in thermal equilibrium:

If an intrinsic semiconductor is in equilibrium, we have $n = p = n_i$

$$np = n_i^2 = N_C N_V \exp\left(\frac{E_V - E_C}{k_B T}\right) = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right),$$

which is independent of the position of the Fermi level and thus valid for doped semiconductors as well. When we denote the position of the Fermi level in the intrinsic material E_{Fi} we may write

$$n_i = N_C \exp\left(\frac{E_{Fi} - E_C}{k_B T}\right) = N_V \exp\left(\frac{E_V - E_{Fi}}{k_B T}\right).$$

which imples:

$$E_{Fi} = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right) = E_C - \frac{E_g}{2} + \frac{k_B T}{2} \ln\left(\frac{N_V}{N_C}\right).$$

Source: textbook

Carrier concentrations

Doped semiconductors:

Under the assumption that the semiconductor is uniformly doped and in equilibrium, a simple relationship between the carrier and dopant concentrations can be established.

$$\rho = q\left(p + N_D^+ - n - N_A^-\right),$$

where q is the elementary charge ($q \approx 1.602 \times 10^{-19}$ C). N_D^+ and N_A^- denote the density of the *ionized donor and acceptor* atoms, respectively.

Considering charge-neutrallity and perfect ionization of all doped atoms.

$$p + N_D - n - N_A = 0,$$

This leads to:

- $n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A} \ll p.$ for p-doped materials ($N_A >> N_D$)
- $p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D} \ll n.$ for n-doped materials ($N_D >> N_A$)

Source: textbook

Carrier concentrations

Doped semiconductors:

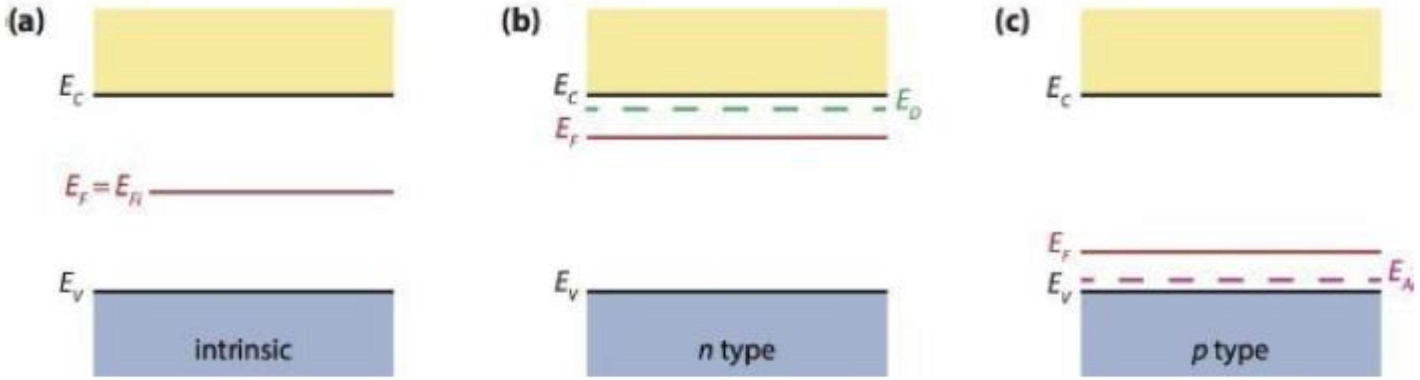


Figure 6.7: A shift of the position of the Fermi energy in the band diagram and the introduction of the allowed energy level into the bandgap due to the doping.

- Doping also influences the position of the Fermi energy. By increasing N_D the Fermi energy will increase. In the *p*-type material the Fermi energy is moved closer to the VB.
- The Fermi level in an *n*-type semiconductor is:

$$E_C - E_F = k_B T \ln\left(\frac{N_C}{N_D}\right) \qquad \text{for } n\text{-type,}$$

$$E_F - E_V = k_B T \ln\left(\frac{N_V}{N_A}\right) \qquad \text{for } p\text{-type.}$$

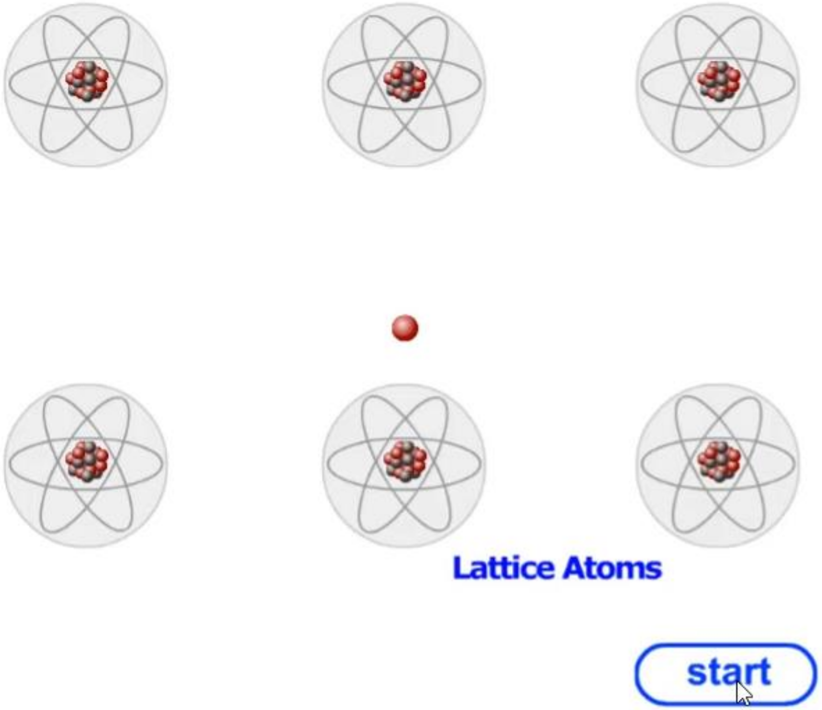
Source: textbook

QUESTIONS ?

Transport properties in semiconductors

Carriers are considered to move „free“

- e^- in E_c and h^+ in E_v are free carriers $\Rightarrow$ can move throughout the lattice
- Carrier moves in random direction for a distance called scattering length before colliding with a lattice atom. After collision, carrier then moves away in a different random direction.
- The velocity of the carriers is determined by the temperature of the lattice
- **no net motion of carriers**, unless there is a concentration gradient or an electric field

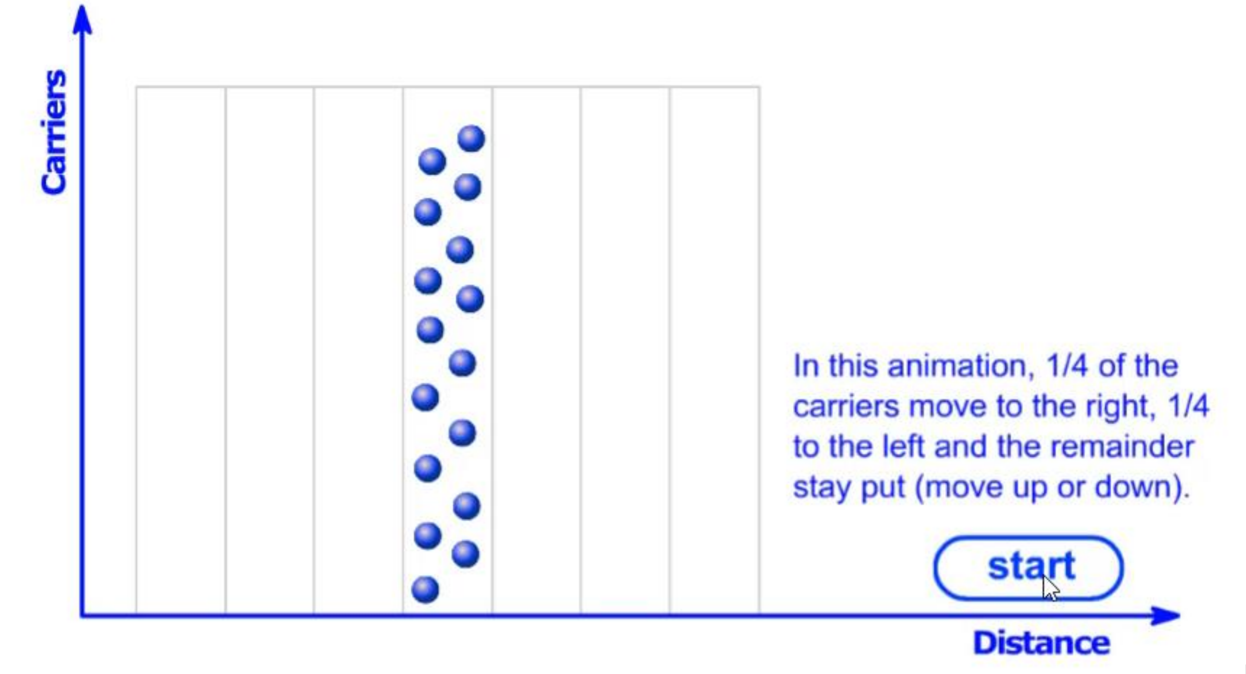


Source: http://www.pveducation.org/pvcdrom/pn-junction/movement-of-carriers-in-semiconductors

Transport properties in semiconductors

Diffusion:

- *Diffusion* is a process whereby particles tend to spread out from regions of high particle concentration into regions of low particle concentration as a result of random thermal motion. The driving force of diffusion is a *gradient* in the particle concentration.
- Rate of diffusion depends on velocity at which carriers move and on the distance between scattering events – termed diffusivity (units: cm^{2}s^{-1})



In this animation, 1/4 of the carriers move to the right, 1/4 to the left and the remainder stay put (move up or down).

Source: textbook

Transport properties in semiconductors

Karlsruher Institut für Technologie

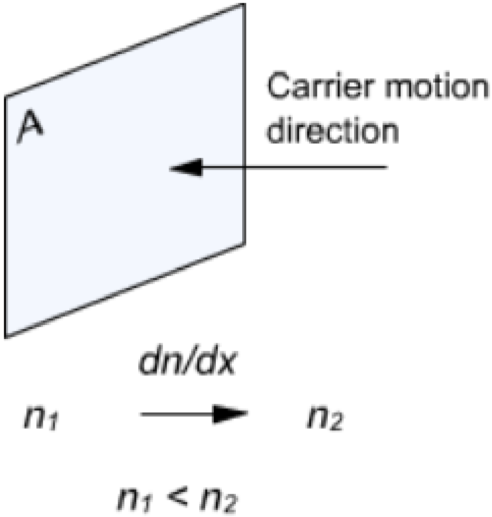
Diffusion:

- Currents resulting from diffusion are proportional to the gradient in particle concentration. For electrons and holes, they are given by:

$$J_{n\_diff} = qD_n \frac{dn}{dx}$$

$$J_{p\_diff} = -qD_p \frac{dp}{dx}$$

$q = 1.602 \times 10^{-19}$ coulombs



With total diffusion current:

$$\mathrm{J_{diff}} = q(D_n \nabla n - D_p \nabla p).$$

- The proportionality constants, D_n and D_p are called the electron and hole *diffusion coefficients*, respectively. They are linked with the mobilities (μ_n, μ_p units: $cm^2V^{-1}s^{-1}$) by the *Einstein relationship:*

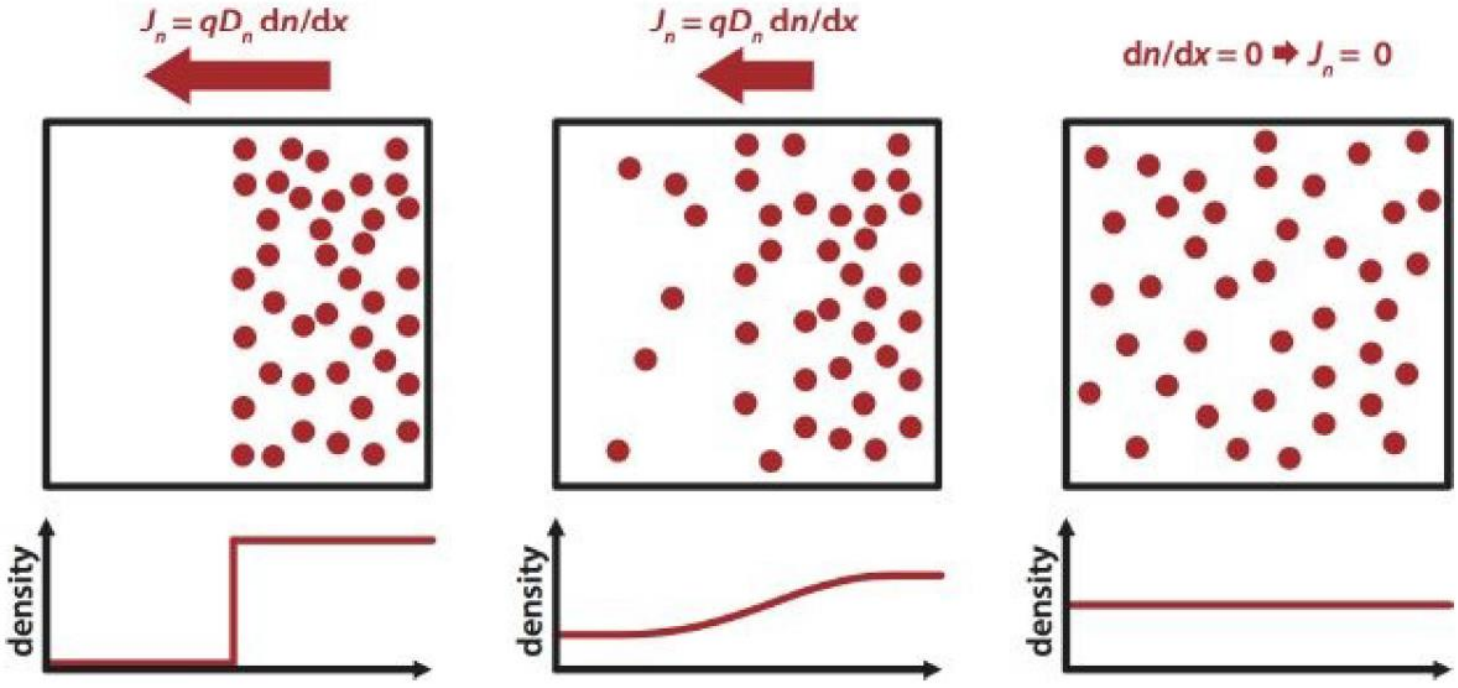
$$D_n = \frac{kT}{q}\mu_n \text{ and } D_p = \frac{kT}{q}\mu_p$$

Source: textbook

Transport properties in semiconductors

Diffusion:

- Visualization of electron diffusion:



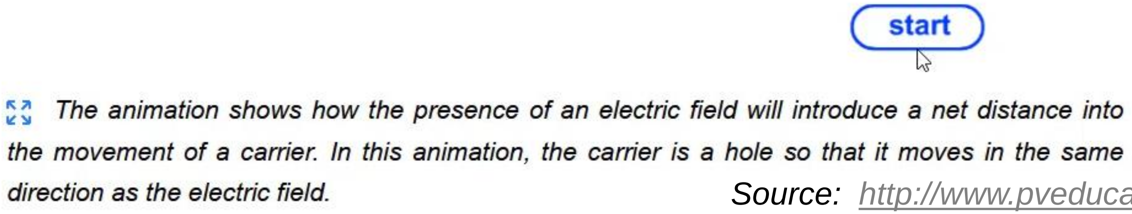
- For example, for c-Si with a doping concentration N_D or N_A at 300 K, the mobilities are $\mu_n \approx 1\,360\ \mathrm{cm^2V^{-1}\,s^{-1}}$., $\mu_p \approx 450\ \mathrm{cm^2V^{-1}\,s^{-1}}$.

Source: textbook

Transport properties in semiconductors

Drift:

- Movement of charge carriers in the presence of an electric field



The animation shows how the presence of an electric field will introduce a net distance into the movement of a carrier. In this animation, the carrier is a hole so that it moves in the same direction as the electric field.

Source: http://www.pveducation.org/pvcdrom/pn-junction/drift

Source: textbook

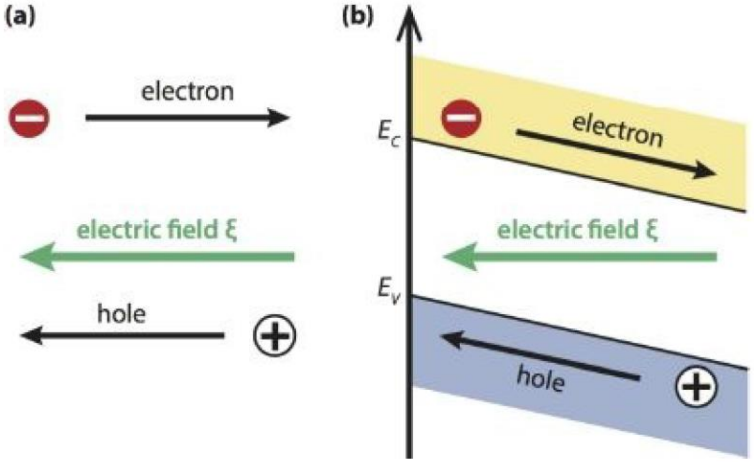
Transport properties in semiconductors

Drift current density is given by:

$$J_{n\_drift} = q\, n\, \mu_n\, E_x \qquad\qquad J_{p\_drift} = q\, p\, \mu_p\, E_x$$

where:

J_x is the current density in the *x*-direction,
E_x (or ξ) - electric field applied in the x-direction,
q – charge on an electron,
n and *p* - electron and hole concentrations,
μ_n and μ_p - electron and hole mobilities



Total current flow is sum of both, hence conductivity σ can be defined as:

$$\sigma = \frac{1}{\rho} = \frac{J_{drift}}{E_x} = q\, n\, \mu_n + q\, p\, \mu_p$$

where ρ is the resistivity (typical unit: Ω.cm)

Source: textbook

Continuity & Movement of Carriers

- Charge is a conserved quantity. Poisson's equation relates the divergence of electric field to the space charge density, ρ :

$$\vec{\nabla}\cdot\vec{D}=\rho \quad => \quad \Delta\varphi(\vec{r})=\rho/\epsilon_0\epsilon(\vec{r})$$

with $\vec{E}=-grad\,(\varphi(\vec{r})), and\ \vec{D}=\epsilon_0\epsilon(\boldsymbol{r})\,\vec{E}$

- Contributions to charge density in semiconductor:
  1) e^- in E_c contribute negative charge, whereas h^+ give positive charge
  2) donor impurity that is ionised, i.e. had extra electron removed, has a net positive charge due to unneutralised positive charge at nucleus ($N_D{}^+$) and vice versa for an ionised acceptor ($N_A{}^-$)
- Combining these contributions:

$$\rho=q\left(p-n+N_D{}^+-N_A{}^-\right)$$

- N.B. most donors and acceptors are ionised under normal conditions so that $N_D{}^+\approx N_D \quad and\ N_A{}^-\approx N_A$

Source: textbook

Continuity & Movement of Carriers

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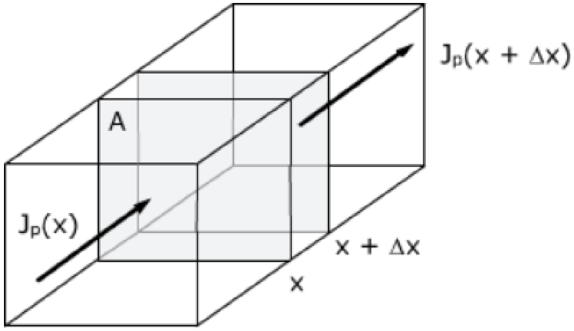
total current densities for *e*$^-$ and *h*$^+$ are the sum of drift and diffusion:

$$J_n = q\, n\, \mu_n\, E_x + qD_n \frac{dn}{dx} \qquad J_p = q\, p\, \mu_p\, E_x - qD_p \frac{dp}{dx}$$

Final "book-keeping" equations to keep track of no. of *e*$^-$ and *h*$^+$ in system, e.g. consider a volume of cross-sectional area *A* and of length Δx



Source: textbook

QUESTIONS ?

Absorption of Light

Photons incident on semiconductor will be either:

1. reflected from the top surface,
2. absorbed in the material or,
3. transmitted through the material.

For PV devices, i) and iii) reflection and transmission are losses
$\Rightarrow$ photons which are not absorbed do not generate power

For absorption, possibility of exciting an e^- from E_v to E_c but depends on energy of photon, E_{ph}:

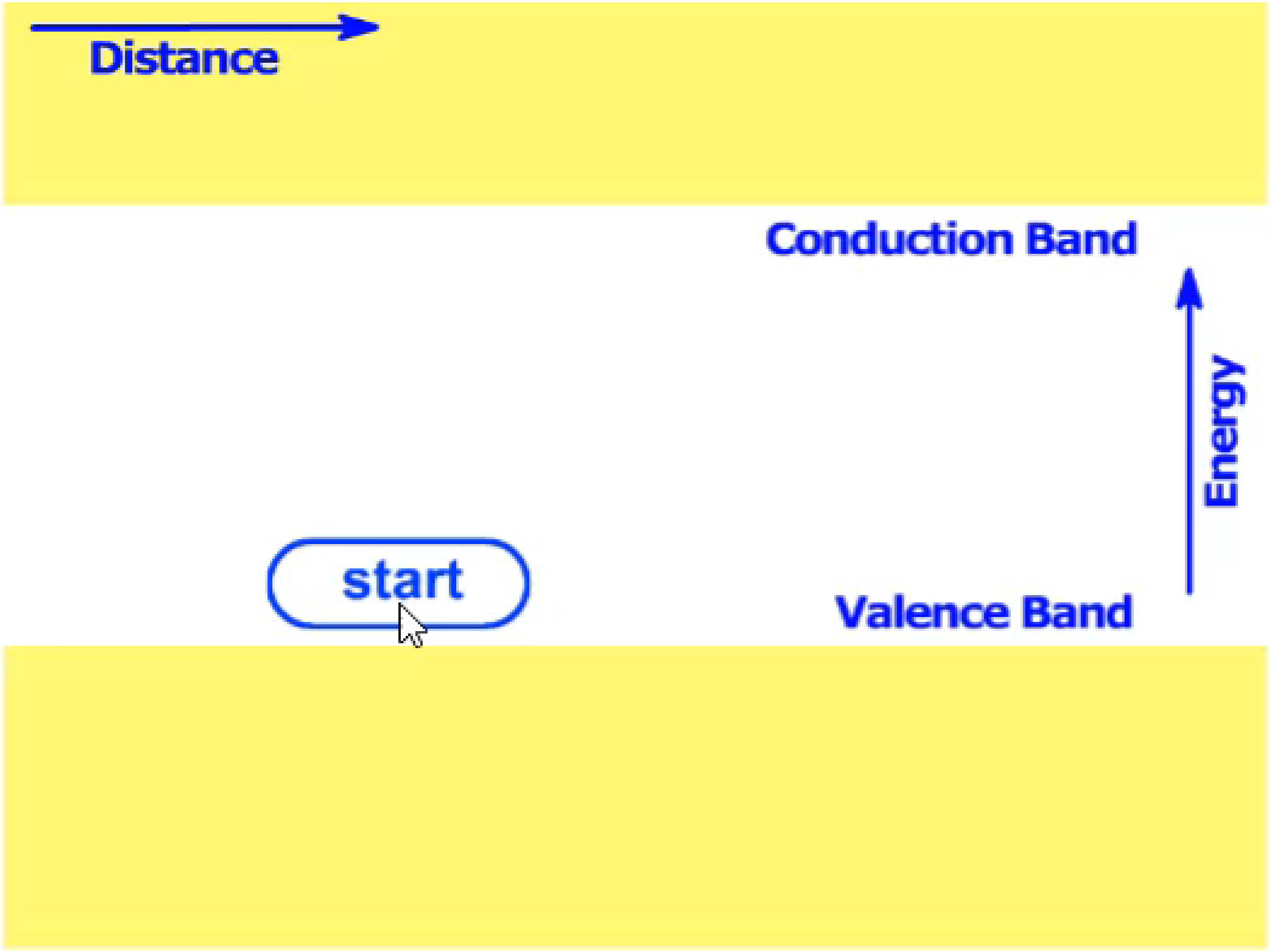
$E_{ph} < E_g$: pass through semiconductor as if it were transparent

$E_{ph} = E_g$: enough energy to create an e^--h^+ pair and are efficiently absorbed

$E_{ph} > E_g$: strongly absorbed, but the excess photon energy is wasted as carriers quickly thermalize back down to the band edges

Absorption of Light

Shown graphically:



Source: *http://www.pveducation.org/pvcdrom/pn-junction/absorption-of-light*

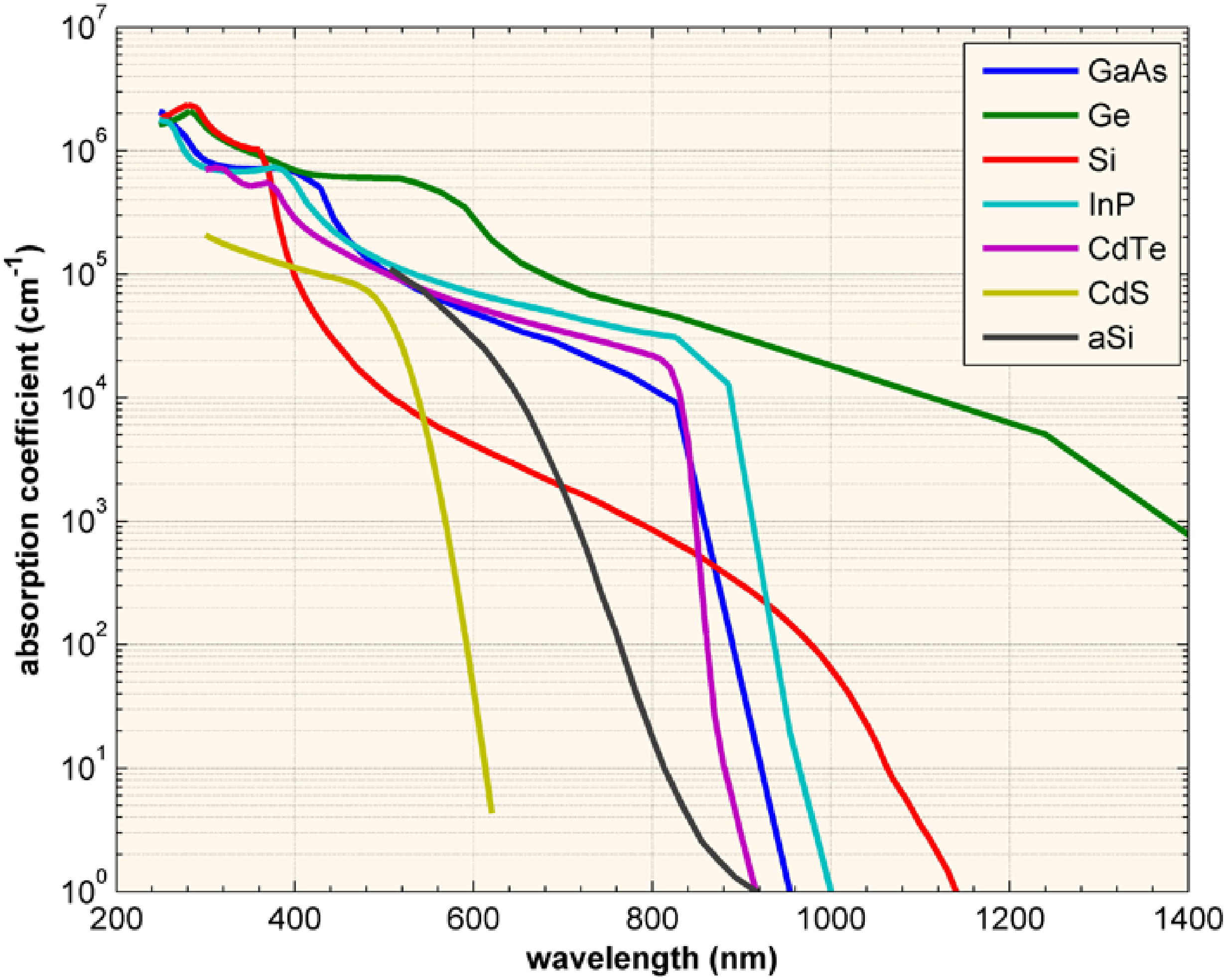
Absorption of Light

Absorption coefficient (α) determines how far into a material light of a particular wavelength can penetrate before it is absorbed.

For photons with energy close to E_g, α is quite low since only those e^- directly at band edge can interact with photon to cause absorption.

At higher energies, more e^- can interact with photon $\Rightarrow$ increased absorption

N.B. for high energy photons α is not constant and is still $\propto \lambda$



Source: http://www.pveducation.org/pvcdrom/pn-junction/absorption-coefficient

Absorption of Light

Neglecting reflection, the amount of light absorbed and the thickness of the absorber correlate. Intensity of light can be calculated according to:

$$I = I_0 \, e^{-\alpha x}$$

where:

α is absorption coefficient (cm^{-1});

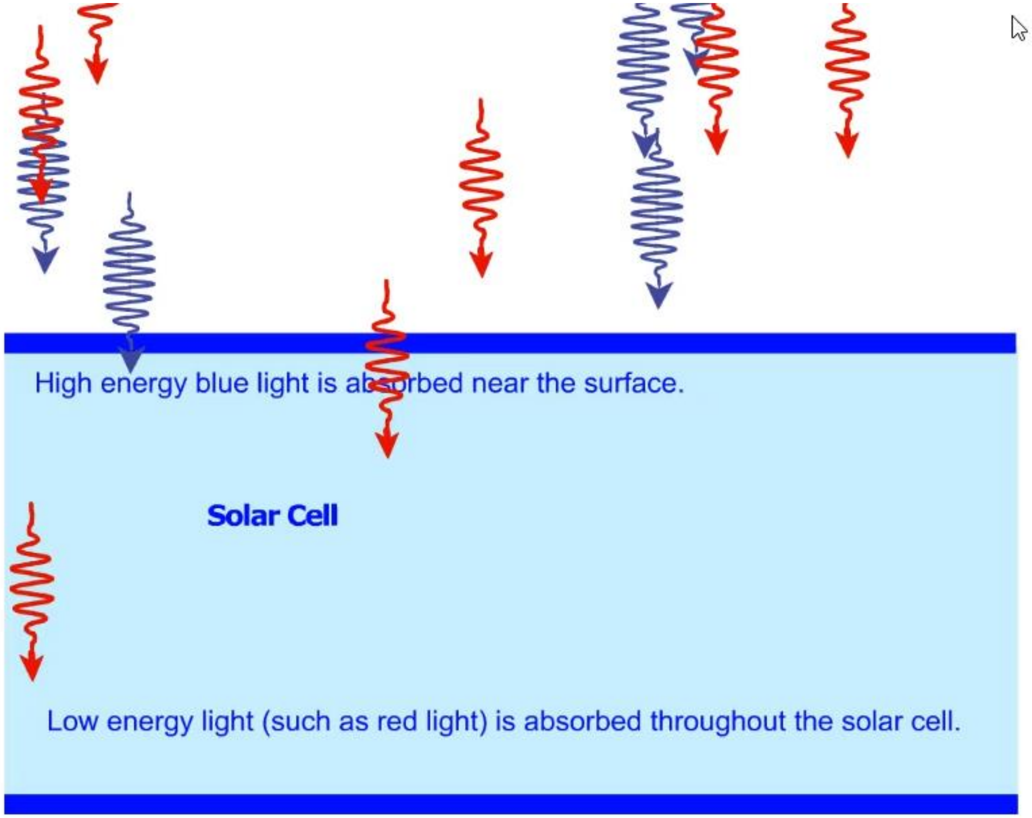
x is the distance into the material; and

I_0 is the light intensity at the top surface

The absorptance A is given by: $\mathrm{A} = 1 - \; I / \, I_0 = \; (1 - e^{-\alpha x})$

Generation rate ≡ number of e^{-} generated at each point in the device due to the absorption of photons

Absorption of Light



The blue photons are absorbed very close to the surface but most of the red photons are absorbed deep in the device.

Source: *http://www.pveducation.org/pvcdrom/pn-junction/absorption-depth*

QUESTIONS ?

Quick Test

- Describe Maxwell‘s Equations and derive harmonic wave solution from Maxwell‘s equations
- What is the wave-particle dualism?
- Explain the Lambert-Beer Law.
- What are the Fresnel Equations?
- Explain teh relation between the complex refractive index, the dielectric function and teh absorption coefficient.
- What is the difference between (i) insulator, (ii) metal, (iii) semiconductor.
- Explain the differences between intrinsic semiconductor, degenerated semiconductor, n-doped semiconductor, and p-doped semiconductor.
- How to calculate the carrier concentrations in the valence band and conduction band? [Explain Fermi-Dirac statistics, Density of States]

Quick Test

- Equation of the concentration of charge carriers in the valence band and the conduction band.
- What is the effective mass of charge carriers?
- Drift vs. diffusion ? Provide the key equations
- Absorption of light in a semiconductor

APPENDIX

Movement of Carriers in Semicond.

Net rate of increase of *e*⁻ **=** rate at which *e*⁻ enter **–** rate at which *e*⁻ exit **+** rate at which they are generated **–** rate at which they recombine

Rates of entering and exiting ∝ current densities, hence:

$$\text{Rate of entering – rate exiting} = \frac{A}{q}\{-J_n(x) - [-J_n(x+\Delta x)]\}$$

$$= \frac{A}{q}\,\frac{dJ_n}{dx}\Delta x$$

$$\text{Rate of generation – rate of recombination} = \frac{A}{q}\,\Delta x\,(G-U)$$

where *G* is the net generation rate under illumination and *U* is the recombination rate. Under steady-state conditions the net rate of increase must be zero such that:

$$\frac{1}{q}\,\frac{dJ_n}{dx}\Delta x = U - G \qquad \text{and} \qquad \frac{1}{q}\,\frac{dJ_p}{dx}\Delta x = -(U-G)$$